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Finite-Strain Fractional-Order Viscoelastic (FOV) Material Models and Numerical Methods for Solving Them

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Fractional-order viscoelastic (FOV) material models have been proposed and studied in 1D since the 1930's, and were extended into three dimensions in the 1970's under the assumption of infinitesimal straining. It was not until 1997 that Drozdov [3] introduced the first finite-strain FOV constitutive equations. In our presentation, we shall continue in this tradition by extending the standard, FOV, fluid and solid, material models introduced in 1971 by Caputo and Mainardi [1] into 3D constitutive formulæ applicable for finite-strain analyses. To achieve this, we generalize both the convected and co-rotational derivatives of tensor fields to fractional order. This is accomplished by defining them first as body tensor fields [4] and then mapping them into space as objective Cartesian tensor fields. Constitutive equations are constructed using both variants for fractional rate, and their responses are contrasted in simple shear.

To aid in the study of such formulæ, and to allow their eventual use in engineering analyses, Diethelm [2] derived general compound quadrature formulæ for solving finite-part integrals. In ensuing papers, we have constructed algorithms that solve fractional-order integrals, derivatives, and differential equations, and we (and others) have studied their existence and uniqueness of solutions, stability, accuracy, and error analysis, along with error refinement using Richardson extrapolation techniques.

After five years of research and development, we now possess a basic suite of numerical tools necessary to study finite-strain FOV constitutive equations and their iterative refinement into a mature collection of material models. Numerical methods still need to be developed for efficiently solving fractional-order integrals, derivatives, and differential equations in a finite element setting where such constitutive formulæ would need to be solved

at each Gauss point in each element of a finite element model, which can number into the millions in today's analyses.

References

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